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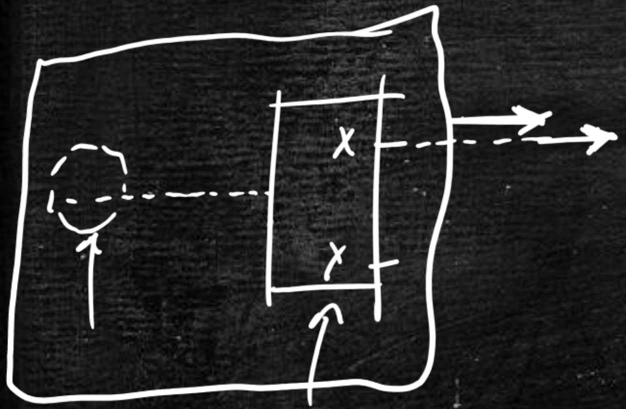
Quantum Theory: Session 03

Quantum Tests

Contents

Review

Review



A. Statistical determinism.

If a quantum system is prepared in such a way that it certainly yields a predictable outcome in a specified maximal test, the various outcomes of **any** other test have definite **probabilities**. In particular, these probabilities do not depend on the details of the procedure used for preparing the quantum system, so that it yields a specific outcome in the given maximal test. A system prepared in such a way is said to be in a **pure** state.

Review

B. Equivalence of maximal tests.

Two maximal tests are equivalent if every preparation that yields a **definite** outcome for one of these tests also yields a **definite** outcome for the other test. In that case, any other preparation (namely one that does not yield a predictable outcome for these tests) will still yield the same **probabilities** for corresponding outcomes of both tests.

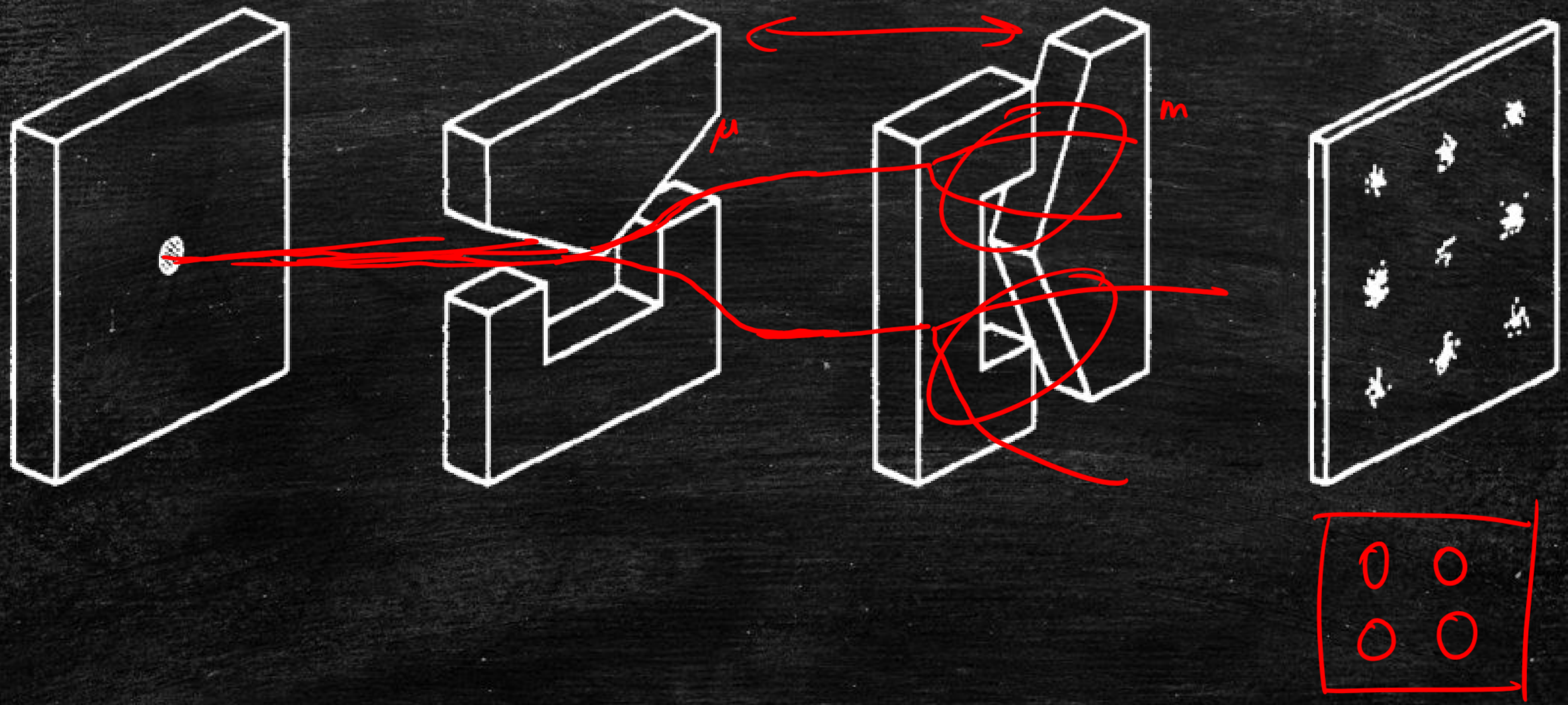
Review

C. Random Mixtures

Quantum systems with N states can be prepared in such a way that every unbiased maximal test has the same probability, N^{-1}, for each one of its outcomes.

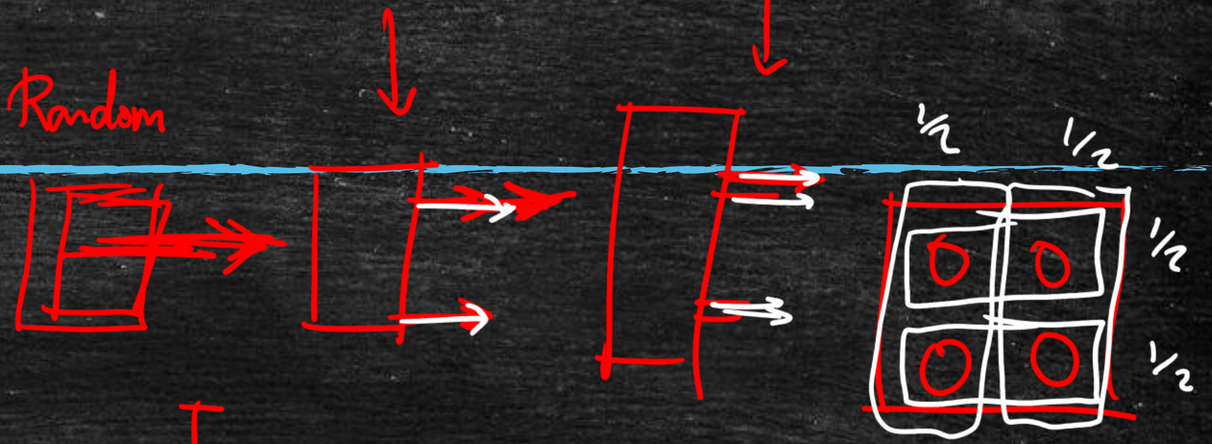
Consecutive Tests

Consecutive Tests



Consecutive Tests

- Stochasticity & Doubly Stochasticity



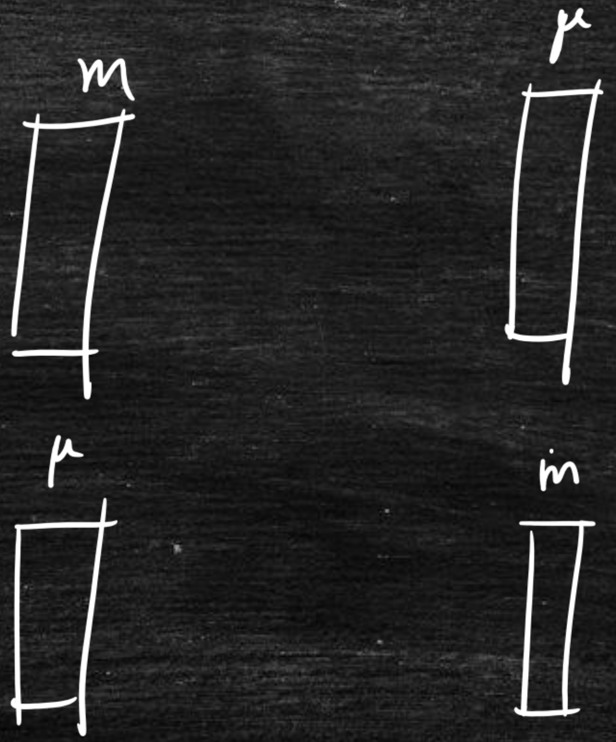
$$I_m \longrightarrow \sum_\mu I_{\mu m} = I_m$$

$$P_{\mu m} = \frac{I_{\mu m}}{I_m} \longrightarrow \sum_\mu P_{\mu m} = 1 \; ; \; \sum_m P_{\mu m} \overset{?}{=} 1$$

$$P'_\mu = \frac{1}{N}, \; P_m = \frac{1}{N} \longrightarrow P'_\mu = \sum_m P_{\mu m} P_m \longrightarrow \sum_m P_{\mu m} = 1$$

Consecutive Tests

- Reversed order



$$P_{\mu m} \xrightarrow{=} \Pi_{m\mu}$$

Consecutive Tests

$$|\langle\phi|\psi\rangle|^2 = |\langle\psi|\phi\rangle|^2 \leftarrow$$

$$\left\{ \begin{array}{ll} S_x^{(P)}, & S_y^{(e)} \\ S_{tot}^2, & S_z \end{array} \right.$$

D. Law of Reciprocity

Let ϕ and ψ denote pure states. Then the probability of observing outcome ϕ in a maximal test following a preparation of state ψ, is equal to the probability of observing outcome ψ in a maximal test following a preparation of state ϕ.

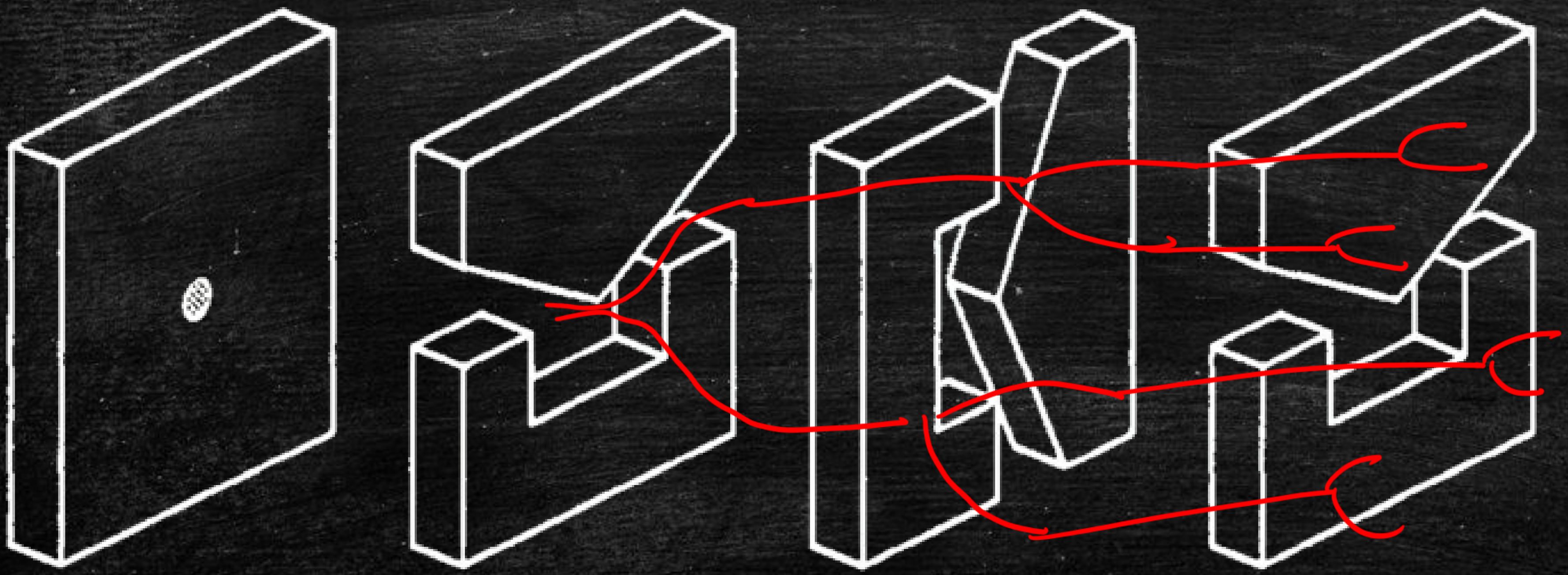
$$P(\phi|\psi) = P(\psi|\phi)$$

A. Lande, Fuondations of Quantum Theory: A Study in Continuity and Symmetry, Yale Univ. Press, New Haven (1955).

The Principle of Interference

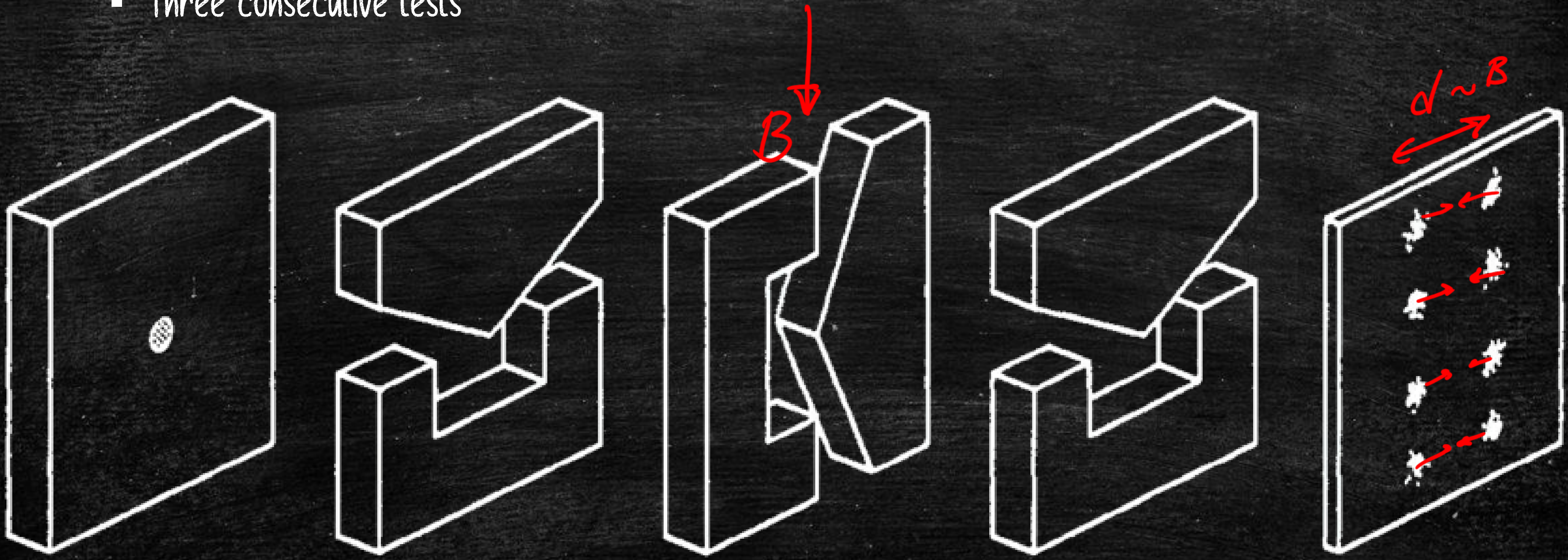
The Principle of Interference

- Three consecutive tests



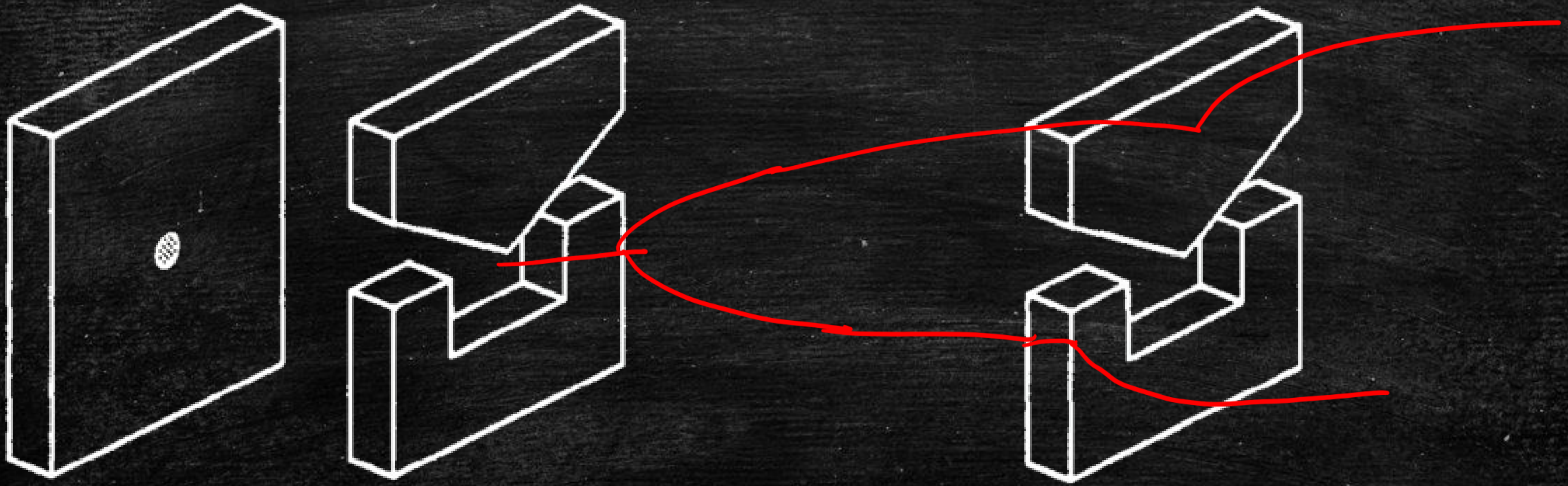
The Principle of Interference

- Three consecutive tests



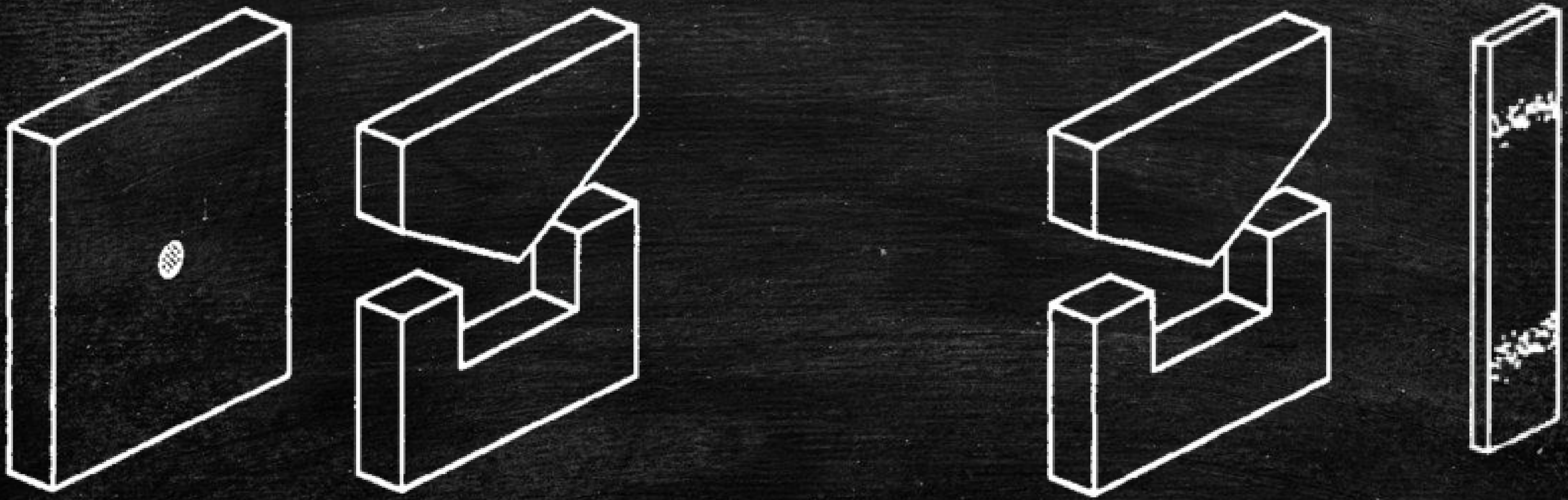
The Principle of Interference

- Three consecutive tests



The Principle of Interference

- Three consecutive tests



The Principle of Interference

- Three consecutive tests

$$m\,\mu\,n: \quad P_{\mu n} P_{\mu m} = \Pi_{n\mu} P_{\mu m}$$

$$\sum_{\mu} P_{\mu n} P_{\mu m} \quad \times$$

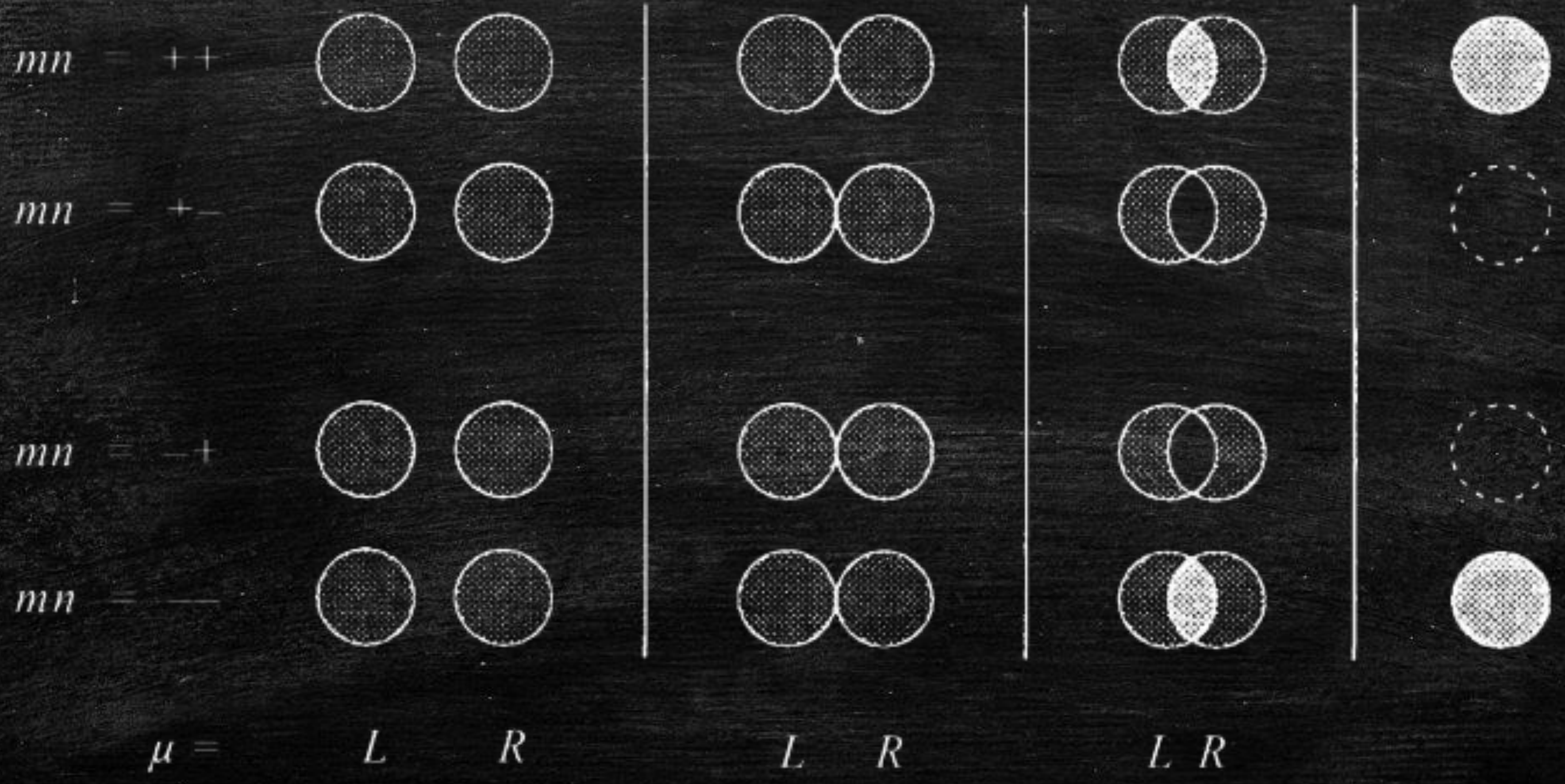
The Principle of Interference

E. Principles of Interference.

If a quantum system can follow several possible paths from a given preparation to a given test, the probability for each outcome of that test is **not** in general the sum of the separate probabilities pertaining to the various paths.

The Principle of Interference

- Three consecutive tests



Transition Amplitudes

Transition Amplitudes

- Classical waves

$\vec{E}$ $\qquad I \propto |E|^2$ $\qquad A, \delta$

$E = A\, e^{ikx - i\omega t} \longrightarrow |A|^2$ $\qquad \Pi_{n\mu}$

$P_{\mu m} = |C_{\mu m}|^2$; $m \mu n$: $P_{\mu n} P_{\mu m}$

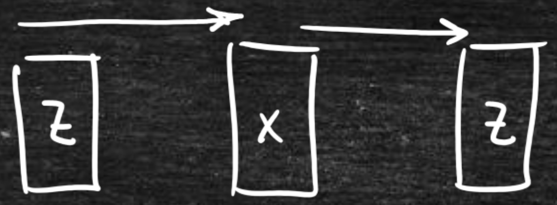
$\Pi_{\mu m} = |\Gamma_{\mu m}|^2$ $\qquad C_{\mu n} C_{\mu m}$

Transition Amplitudes

F. Law of Composition of Transition Amplitudes.

The phases of the transition amplitudes can be chosen in such a way that, if several paths are available from the initial state to the final outcome, and if the dynamical process leaves no trace allowing to distinguish which path was taken, the complete amplitude for the final outcome is the **sum** of the amplitudes for the various paths.

Transition Amplitudes



- Consistency

$$m \mu n : \quad \Gamma_{n\mu} C_{\mu m}$$

$$\sum_{\mu} \Gamma_{n\mu} C_{\mu m} = \delta_{nm} \quad ; \quad |\Gamma_{n\mu}| = |C_{\mu m}|$$

$$\Gamma_{n\mu} = C_{\mu n}^{*} \hookrightarrow \sum_{\mu} C_{\mu n}^{*} C_{\mu m} = \delta_{nm} \longrightarrow \underbrace{C^{\dagger} C = \mathbb{1}}_{\text{unitary}}$$

Transition Amplitudes: Determination of Phases of Transition Amplitudes

$$\sum_{\mu} C_{\mu n}^{*} C_{\mu m} = \delta_{nm} \quad ; \quad C_{\mu m}$$

$\sum_{\mu} |C_{\mu n}|^2 = 1$

$|C_{\mu m}| = \sqrt{P_{\mu m}}$

$C^{\dagger} C = 1\!\!1$ ($N \times N$)

$nm \to mn$

$m = n$

N : # of Phases : N^2

$1 + 2 + 3 + \dots + N-1 = \frac{(N-1)N}{2} \times 2 = N(N-1)$

$\sum_{n} |C_{\mu n}|^2 = 1 \Rightarrow N-1 \longrightarrow N(N-1) - (N-1) = (N-1)^2 \neq N^2$

$\sum P_{\mu} = 1$

$C_{\mu 1}, \; C_{1\mu}$