Composite systems

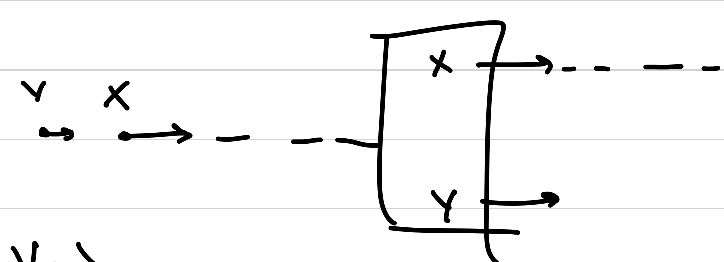
Prepared independently: $e: |u\rangle \longrightarrow u_e(\vec{r}_e)$

$$p: |v\rangle \longrightarrow v_p(\vec{r}_p)$$

$$u_m \quad , v_\mu$$

$$|w\rangle = |u\rangle \otimes |v\rangle \implies W_{\underbrace{m\mu}} = u_m v_\mu$$

u_1, u_2 $\qquad v_1, v_2$

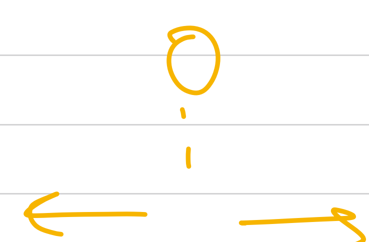


$$|w\rangle = \alpha |u_1\rangle \otimes |v_1\rangle + \beta |u_2\rangle \otimes |v_2\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\underbrace{\left(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle\right)} \xrightarrow{\text{measurement}} \begin{matrix} 50\% & |\uparrow\rangle \otimes |\downarrow\rangle \\ 50\% & |\downarrow\rangle \otimes |\uparrow\rangle \end{matrix}$$

$$|\psi\rangle : \underbrace{50\% \rightarrow |\uparrow\rangle \otimes |\downarrow\rangle}$$

$$\underbrace{50\% \rightarrow |\downarrow\rangle \otimes |\uparrow\rangle}$$



$$\hat{\rho} = \frac{1}{2}\left(\hat{\mathbb{1}} + \sum_j a_j \hat{\sigma}_j\right) \quad ; \quad a_j = \mathrm{Tr}\left[\hat{\rho}\,\hat{\sigma}_j\right]$$

$$P_i \rightarrow |\psi_i\rangle \implies \hat{\rho} = \sum_i P_i |\psi_i\rangle\langle\psi_i|$$

1'st Case: $|\psi\rangle = \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)\frac{1}{\sqrt{2}}$

$$\hat{\rho} = \frac{1}{2}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)\left(\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right)$$

$$= \frac{1}{2}\left(|\uparrow\downarrow\rangle\langle\uparrow\downarrow| + |\downarrow\uparrow\rangle\langle\downarrow\uparrow| - |\downarrow\uparrow\rangle\langle\uparrow\downarrow| - |\uparrow\downarrow\rangle\langle\downarrow\uparrow|\right)$$

$|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$

$$\rho_1 = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

2nd Case: $\hat{\rho} = \frac{1}{2}|\uparrow\downarrow\rangle\langle\uparrow\downarrow| + \frac{1}{2}|\downarrow\uparrow\rangle\langle\downarrow\uparrow|$

$$\hookrightarrow \rho_2 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \frac{\hat{\mathbb{1}}}{2}$$

$$\hat{\rho}_1 = \hat{\rho}_2 - \frac{\hat{\sigma}_x}{2}$$

$\hat{\rho}_2 = \frac{1}{2}\hat{\mathbb{1}} \longrightarrow a_j = \mathrm{Tr}[\hat{\rho}\hat{\sigma}_j] = \frac{1}{2}\mathrm{Tr}[\sigma_j] = 0$

$$\hat{\rho}_1 = \hat{\rho}_2 - \frac{\hat{\sigma}_x}{2} \Rightarrow a_j = \mathrm{Tr}[\hat{\rho}\hat{\sigma}_j] = \underbrace{\mathrm{Tr}[\hat{\rho}_2\sigma_j]}_{0} - \frac{1}{2}\mathrm{Tr}[\hat{\sigma}_x\hat{\sigma}_j]$$

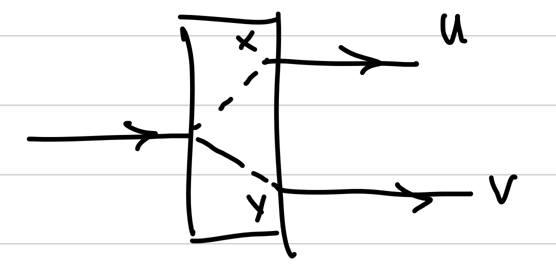
$a_y = 0$

$a_z = 0$

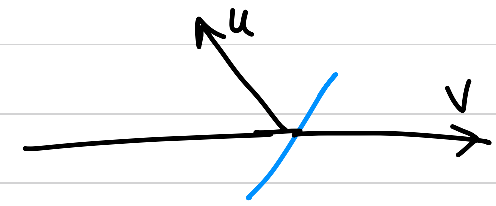
$a_x = -\frac{1}{2}\mathrm{Tr}[\sigma_x^2] = -\frac{1}{2}\mathrm{Tr}[\hat{\mathbb{1}}]$

$= -1$

an example : $H = \dfrac{P_1^2+P_2^2}{2m} + V(x_1, x_2)$



Polarization : $|X\rangle$, $|Y\rangle$



Path : $|u\rangle$, $|v\rangle$

$$\mathcal{H} = \text{span}\{ |X\rangle \otimes |u\rangle , |X\rangle \otimes |v\rangle , |Y\rangle \otimes |u\rangle , |Y\rangle \otimes |v\rangle \}$$

$|X\rangle \otimes |u\rangle$:

$$\left(\alpha|X\rangle + \beta|Y\rangle\right) \otimes |W\rangle$$

$$\longrightarrow \alpha|X\rangle \otimes |u\rangle + \beta|Y\rangle \otimes |v\rangle$$