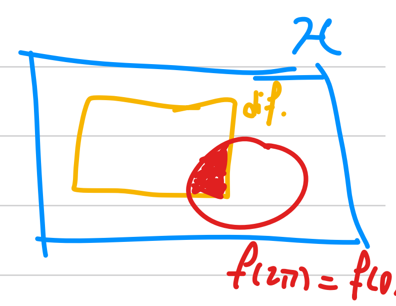
$$(P - \langle P \rangle)\psi = i m\omega (X - \langle x \rangle)\psi$$

$$-i\hbar \psi' = \langle p \rangle \psi + i m\omega (X - \langle x \rangle)\psi$$

$$\ln \psi = i \frac{\langle p \rangle}{\hbar} x - \frac{m\omega}{2\hbar} x^2 + \frac{m\omega}{\hbar} \langle x \rangle x$$

$$\underbrace{\psi = C \, e^{i \frac{\langle p \rangle}{\hbar} x} \, e^{-\frac{m\omega}{2\hbar} x^2 + \frac{m\omega}{\hbar} \langle x \rangle x}}$$

$$\mathcal{H} = \mathcal{L}^2([0, 2\pi]) \quad , \quad \langle v | u \rangle = \int_0^{2\pi} v^*(x)\, u(x)\, dx$$



$$\hat{P} = -i\hbar \, d/dx \longrightarrow D_P = \text{dif.} \mid f(2\pi) = f(0)$$

$$\hat{X} = x$$

$$P X u_m$$

$$f(x) = x\, u_m(x)$$

$$f(2\pi) \quad \sqrt{2\pi} \neq 0$$

$$\Delta X \, \Delta P \overset{?}{\geq} \frac{\hbar}{2} \qquad \times$$

$$u_m(x) = \frac{1}{\sqrt{2\pi}} e^{imx} \longrightarrow \Delta P = 0$$

$$\hookrightarrow \Delta X = \frac{\pi}{\sqrt{3}} \simeq 1.8$$

$$\Delta X \, \Delta P = 0$$

$$[A, BC] = [A, B] C + B [A, C]$$

$$\left. \begin{array}{l} [A, B] = 0 \\ [A, C] = 0 \end{array} \right\} \Longrightarrow [A, BC] = 0 \longrightarrow \text{finite matrices}$$

$$\mathcal{H} = \mathcal{L}^2(\mathbb{R}) \quad ; \quad \langle u|v\rangle = \int_{-\infty}^{\infty} u^* v \, dx$$

$$A = \frac{x}{|x|} \quad , \quad B = \frac{1}{x} \quad , \quad C = x \, d/dx$$

$$AB = BA = \frac{1}{|x|} \longrightarrow [A,B] = 0$$

$$AC = \underbrace{|x| \, d/dx}_{\hat{D}} \quad ; \quad CA = \underbrace{x \, d/dx\left(\frac{x}{|x|}\right)}_{2x\,\delta(x)} + \underbrace{|x| \frac{d}{dx}}$$

$\hat{D}\psi = 0 \ \forall \psi \in \mathcal{H}$

$\underbrace{x\,\delta(x)}_{\hat{D}}\psi(x) = 0$ iff $\psi(x)$ is less singular than $\frac{1}{x}$

$\psi \in \mathcal{H} = \mathcal{L}^2(\mathbb{R}) \longrightarrow \psi(x)$ " " than $\frac{1}{\sqrt{x}}$

$\Big\}$ $x\,\delta(x) = 0$

$$AC = CA \longrightarrow [A,C] = 0$$

$$BC = d/dx \quad , \quad A = \frac{x}{|x|} \longrightarrow [A,BC]\psi = \frac{x}{|x|}\frac{d}{dx}\psi - \frac{d}{dx}\left(\frac{|x|}{x}\psi\right)$$

$$= -\frac{d}{dx}\left(\frac{|x|}{x}\right)\psi$$

$$[A,BC] = -2\delta(x) \neq 0$$

$x\,\delta(x)$

$\underset{1/x}{\underset{\downarrow}{B}}\psi \longrightarrow$ more singular than $\frac{1}{x}$

$$\underset{1/x}{\underset{\downarrow}{B}} \ \overset{x\delta(x)}{\widetilde{[A,C]}}$$

$\hat{D}(s)\,\psi(x) = s\,\psi(s^2 x) \longrightarrow$ linear & unitary

$[A, BD] = 0 \quad \forall s$ $\qquad$ $D(s_1) - D(s_2)$

$$\hat{D}'(1) = \lim_{\varepsilon \to 0}\left(\frac{\hat{D}(1+\varepsilon) - \hat{D}(1)}{\varepsilon}\right)$$

$[A, BD'(1)] \neq 0$

Truncation : $\quad \mathcal{H} = L^2([-1,1]) \qquad \langle u | v \rangle = \int_{-1}^{1} u^* v \, dx$

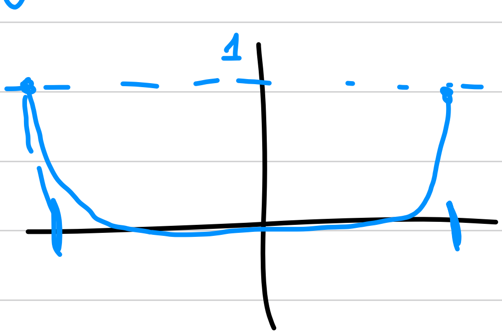
$\hat{x}\,\psi(x) = x\psi(x) \longrightarrow D_{\hat{x}} = \mathcal{H}$

$x\psi(x) = \xi\psi(x) \longrightarrow (x-\xi)\psi(x) = 0 \Longrightarrow \psi(x) = 0 \ \forall\, x \neq \xi$

$\Longrightarrow \psi(x) = \delta(x) \notin \mathcal{H}$

$\sqrt{\delta(x)} \longrightarrow$ Doesn't exist !

x^{100}



$\mathcal{H} \longrightarrow N$ dim $\qquad N \gg 1$

ex. $\quad f(x) \longrightarrow \mathbb{P}_N(x) \quad ; \quad N \gg 1$

$\mathbb{P}_N(x - x_0)$

$\mathcal{H}' = \mathbb{P}_N([-1,1]) \longrightarrow x \quad x^N = x^{N+1} \notin \mathcal{H}'$

$$X_{mn} = \langle m|\hat{X}|n\rangle \quad ; \quad u_n(X) = \sqrt{n+\tfrac{1}{2}}\, P_n(X) \quad ; n \in \{0, \dots, N\}$$

$$(2n+1)\, x P_n(x) = n\, P_{n-1}(x) + (n+1)\, P_{n+1}(x)$$

$$(2n+1)\int_{-1}^{1} x\, P_n(x) P_m(x)\, dx = \left(n\, \delta_{n-1,m} + m\, \delta_{n+1,m}\right) \frac{1}{m+\tfrac{1}{2}}$$

$$X_{nm} = \int_{-1}^{1} u_m\, x\, u_n\, dx = \left(m\, \delta_{m,n+1} + n\, \delta_{n,m+1}\right) \frac{1}{\sqrt{(2m+1)(2n+1)}}$$

$m, n \ll N$

$$\sum_n X_{mn} v_n = s\, v_m$$

$$X = \begin{pmatrix} 0 & \ddots & & 0 \\ \ddots & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ 0 & & \ddots & \ddots \end{pmatrix}$$

$$v(x) = \sum_n v_n u_n(x)$$

$$\hat{D}(s)\, \psi(x) = s\, \psi(s^2 x) \longrightarrow \text{linear \& unitary}$$

$$[A, BD] = 0 \quad \forall s$$

چیزهای اضافه

$$\hat{D}'(1) = \lim_{\varepsilon \to 0} \left(\frac{\hat{D}(1+\varepsilon) - \hat{D}(1)}{\varepsilon} \right)$$

$$[A, B D'(1)] \neq 0$$

$$D(s)\,\psi(x) = s\,\psi(s^2 x)$$

linearity: $D(s)\big(a\psi(x) + b\,\phi(x)\big) = s\big(a\psi(s^2x) + b\,\phi(s^2x)\big)$

$$= a\big(s\,\psi(s^2x)\big) + b\,\big(s\,\phi(s^2x)\big)$$

$$= a\,\hat{D}\psi + b\,\hat{D}\phi$$

unitary: $\langle\psi|\phi\rangle = \langle\hat{D}\psi|\hat{D}\phi\rangle$

$$= \int s\,\psi(s^2x)^*\; s\,\phi(s^2x)\, dx = \int \psi^*(u)\,\phi(u)\, du$$

$$u := s^2 x$$

$$\frac{x}{|x|}\phi(x)$$

$A = \frac{x}{|x|}$, $B = \frac{1}{x}$; $[A, BD]\,\psi = ABD\psi - BDA\psi = 0$ ✓

$$\underbrace{ABD\psi = \frac{1}{|x|}\, s\,\psi(s^2x)}\ ;\ \underbrace{BDA\psi = \frac{1}{x}\, s\,\frac{\cancel{s^2}x}{|\cancel{s^2}x|}\,\psi(s^2x) = s\,\frac{1}{|x|}\,\psi(s^2x)}$$

$$D\psi = s\,\psi(s^2x) \longrightarrow \hat{D}'(1)\,\psi = \lim_{\varepsilon\to 0}\left(\frac{\hat{D}(1+\varepsilon)\psi - \hat{D}(1)\psi}{\varepsilon}\right)$$

$$\hat{D}'(1)\,\psi = \lim_{\varepsilon\to 0}\left(\frac{(1+\varepsilon)\,\psi(x+2\varepsilon x) - \psi}{\varepsilon}\right)$$

$$(1+\varepsilon)\,\psi(x+2\varepsilon x) = (1+\varepsilon)\big(\psi(x) + \psi'(x)\; 2\varepsilon x\big) = \cancel{\psi(x)} + \varepsilon\psi(x) + 2x\psi'(x)\varepsilon$$

$$\hat{D}'(1)\,\phi(x) = \psi(x) + 2x\,\psi'(x) \longrightarrow \underbrace{\hat{D}'(1) = \hat{1} + 2x\, d/dx}$$

$$A = \frac{x}{|x|} \quad , \quad B = 1/x$$

$$[A, B\hat{D}'(1)]\psi = ABD'\psi - BD'A\psi$$

$$\frac{x}{|x|}\frac{1}{x}(\psi + 2x\psi') = \frac{\psi + 2x\psi'}{|x|}$$

$$BD'A\psi = \frac{1}{x}\left(\mathbb{1} + 2x\frac{d}{dx}\right)\left(\frac{x}{|x|}\psi\right) = \frac{\psi}{|x|} + 2\frac{d}{dx}\left(\frac{x}{|x|}\psi\right)$$

$$= \underline{\frac{\psi}{|x|}} + \underline{2\delta(x)\psi(x)} + \underline{2\frac{x}{|x|}\psi'}$$

$$[A, B\, D(s)] = 0 \quad \forall\, s$$

$$D'(1) = D(1+\varepsilon) - D(1) \rightarrow [A, B(D(1+\varepsilon) - D(1))] = [A, BD(1+\varepsilon)]$$

$$- [A, BD(1)]$$