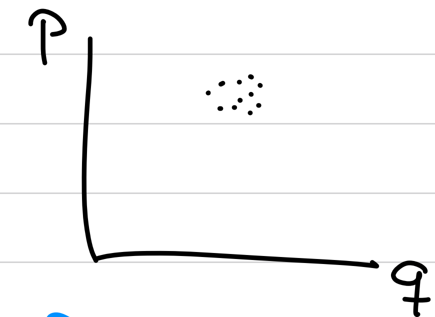
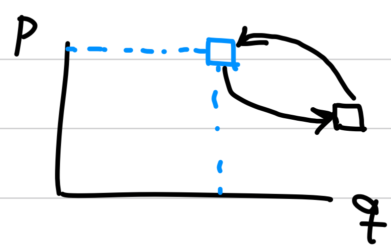
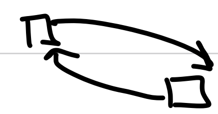
Review: $$\frac{\partial f_1}{\partial t} = \{H_1, f_1\} + \left(\frac{\partial f_1}{\partial t}\right)_{coll.} \qquad V=0\ , H_1 = \frac{p^2}{2m}$$

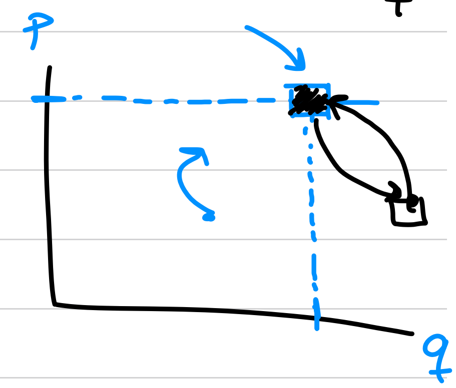


Equilibrium: $\frac{\partial f_1}{\partial t} = 0$; if $\{H_1, f_1\} = 0 \Longrightarrow \left(\frac{\partial f_1}{\partial t}\right)_{col.} = 0$

Detailed Balance :







$$f_1(\vec{r},\vec{p}_1{}')\, f_1(\vec{r},\vec{p}_2{}') = f_1(\vec{r},\vec{p})\, f_1(\vec{r},\vec{p}_2)$$

$$F(x_1+x_2) = F(x_1) + F(x_2) \quad \forall\, x_1, x_2$$

$$F'(x_1+x_2) = F'(x_1) \longrightarrow F'(x) = \text{Const.} \longrightarrow \text{linear}$$

$$\ln f_1(\vec{r},\vec{p}) = \beta\left(\mu - E(\vec{p}) + \vec{u}\cdot\vec{p}\right)$$

$$E(p) = p^2/2m \ : \quad f_1(\vec{r},\vec{p}) = \overbrace{\frac{N}{V}}^{n}\left(\frac{\beta}{2\pi m}\right)^{3/2} e^{-\beta\frac{m}{2}(\vec{v}-\vec{u})^2} \Longrightarrow \text{Boltzmann distribution}$$

$$\text{if } \{H_1, f_1\} \neq 0 \longrightarrow f_1(\vec{r},\vec{p}) \Longrightarrow \begin{cases} \beta(\vec{r}) \\ \vec{u}(\vec{r}) \ ,\ n(\vec{r}) \end{cases}$$

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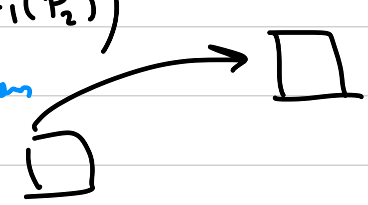
a brief look on it's Quantum Version : $f_1(\vec{p})$: Average occupation # of state $\vec{p}$

$$\text{Rate} = \omega(\vec{p},\vec{p}_2 \,|\, \vec{p}_1{}',\vec{p}_2{}')\, f_1(\vec{p})\, f_1(\vec{p}_2)\,\left(1 \pm f_1(\vec{p}_1{}')\right)\left(1 \underset{\text{Fermions}}{\overset{\text{Bosons}}{\pm}} f_1(\vec{p}_2{}')\right)$$



$$\ln\left(\frac{f_1}{1 \pm f_1}\right) = \beta\left(\mu - E(\vec{p}) + \vec{u}\cdot\vec{p}\right)$$

$$\longrightarrow \boxed{f_1(\vec{p}) = \frac{1}{e^{-\beta(\mu - E(\vec{p}) + \vec{u}\cdot\vec{p})} \mp 1}}$$

Bose – Einstein distribution
Fermi – Dirac

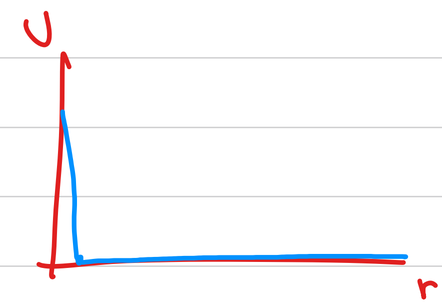
Boltzmann Equation again ! $V=0$;

1'st BBGky :

$$\left(\frac{\partial}{\partial t}+\frac{\vec p_1}{m}\cdot\frac{\partial}{\partial \vec r_1}\right)f_1=\int d^3\vec r_2\, d^3\vec p_2\ \frac{\partial U(\vec r_1-\vec r_2)}{\partial \vec r_1}\cdot\frac{\partial f_2}{\partial \vec p_1}$$

2'nd BBGky :

$$\left(\frac{\partial}{\partial t}+\frac{\vec p_1}{m}\cdot\frac{\partial}{\partial \vec r_1}+\frac{\vec p_2}{m}\cdot\frac{\partial}{\partial \vec r_2}-\frac{1}{2}\frac{\partial U(\vec r_1-\vec r_2)}{\partial \vec r_1}\cdot\left[\frac{\partial}{\partial \vec p_1}-\frac{\partial}{\partial \vec p_2}\right]\right)f_2=\int d^3\vec r_3\, d^3\vec p_3\left(\frac{\partial U(\vec r_1-\vec r_3)}{\partial \vec r_1}\cdot\frac{\partial}{\partial \vec p_1}+\frac{\partial U(\vec r_2-\vec r_3)}{\partial \vec r_2}\cdot\frac{\partial}{\partial \vec p_2}\right)f_3$$



$$* \quad f_2(\vec r_1,\vec r_2,\vec p_1,\vec p_2)=f_1(\vec r_1,\vec p_1)\,f_1(\vec r_2,\vec p_2) \quad *$$

$$\int f_3\ d^3\vec r_3\, d^3\vec p_3 = (N-2)\,f_2 \sim N f_2$$

$N\sim 10^{23}$

$$\left\langle\frac{\partial U}{\partial r}\right\rangle \underset{|\delta \vec r|\sim d}{\frac{\partial}{\partial p}}\int d^3\vec r_3\, d^3\vec p_3\ f_3 \sim \left\langle\frac{\partial U}{\partial r}\,\frac{\partial f_2}{\partial p}\right\rangle \boxed{N\,\frac{d^3}{V}}$$

$d\sim 10^{-9}$

$V\sim 1$

$\hookrightarrow \sim 10^{-3}-10^{-4}$

$$\left(\cancel{\frac{\partial}{\partial t}}+\frac{\vec p_1}{m}\cdot\frac{\partial}{\partial \vec r_1}+\frac{\vec p_2}{m}\cdot\frac{\partial}{\partial \vec r_2}-\frac{1}{2}\frac{\partial U(\vec r_1-\vec r_2)}{\partial \vec r_1}\cdot\left[\frac{\partial}{\partial \vec p_1}-\frac{\partial}{\partial \vec p_2}\right]\right)f_2\simeq 0$$

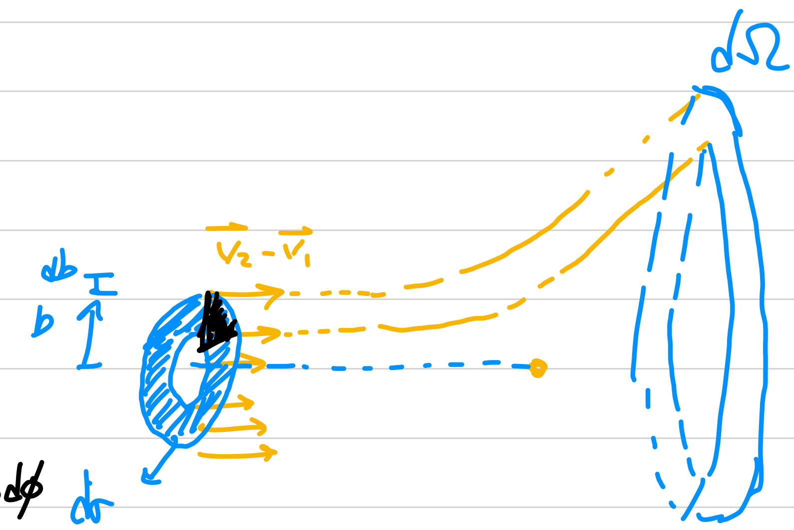
$$f_2(\vec r_1,\vec r_2,\vec p_1,\vec p_2)\longrightarrow\left\{\begin{array}{ll}\vec r:=\vec r_2-\vec r_1 & ;\ \vec p:=\dfrac{\vec p_1-\vec p_2}{2}\\[2ex] \vec R:=\dfrac{\vec r_1+\vec r_2}{2} & ,\ \vec P:=\vec p_1+\vec p_2\end{array}\right\}\Rightarrow f_2(\vec r,\vec R,\vec p,\vec P)$$

$$\left(\frac{\vec p}{m}\cdot\frac{\partial}{\partial \vec r}-\frac{\partial U(\vec r)}{\partial \vec r}\cdot\frac{\partial}{\partial \vec p}\right)f_2=0 \longrightarrow \underline{\underline{|\vec r_1-\vec r_2|\sim d}}$$

$$\left(\frac{\partial f_1}{\partial t}\right)_{\text{coll.}}=\int d^3\vec r_2\, d^3\vec p_2\ \frac{\partial U(\vec r_1-\vec r_2)}{\partial \vec r_1}\cdot\frac{\partial f_2}{\partial \vec p_1}=\int d^3\vec r_2\, d^3\vec p_2\ \frac{\partial U}{\partial \vec r}\cdot\left(\frac{\partial}{\partial \vec p_1}-\frac{\partial}{\partial \vec p_2}\right)f_2$$

$$\left(\frac{\partial f_1}{\partial t}\right)_{coll.} = \int d^3\vec{r}_2\, d^3\vec{p}_2\, \frac{\partial U}{\partial \vec{r}} \cdot \frac{\partial f_2}{\partial \vec{p}} = \int_{|\vec{r}| \le d} \frac{\vec{p}}{m} \cdot \frac{\partial f_2}{\partial \vec{r}}\, d^3\vec{r}_2\, d^3\vec{p}_2$$

Scattering Cross Section:



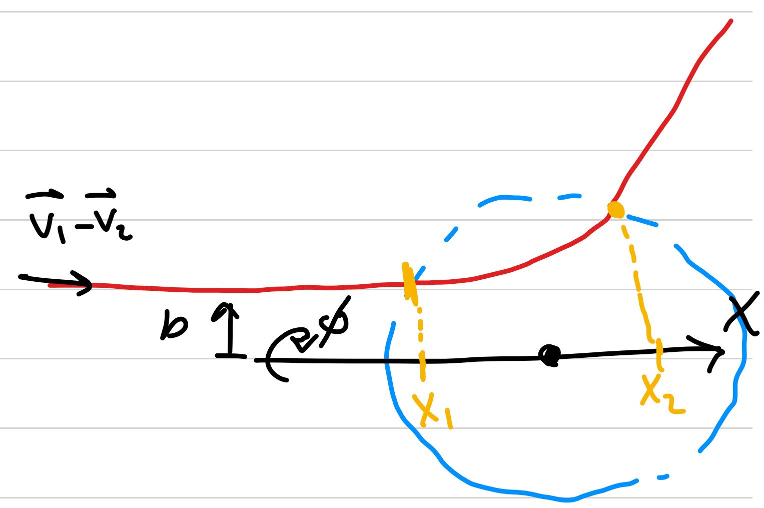
$$d\sigma = b\, db\, d\phi$$

$$I = \frac{N}{V} |\vec{V}_2 - \vec{V}_1| \;;\; d\sigma = \left|\frac{d\sigma}{d\Omega}\right| d\Omega = b\, db\, d\phi$$

$$I\, d\sigma = \frac{N}{V} |\vec{V}_2 - \vec{V}_1| \left|\frac{d\sigma}{d\Omega}\right| d\Omega = \frac{N}{V} |\vec{V}_2 - \vec{V}_1|\, b\, db\, d\phi$$

$$\omega(\vec{P}, \vec{P}_2 | \vec{P}_1', \vec{P}_2')\, d^3\vec{P}_1'\, d^3\vec{P}_2' = |\vec{V}_2 - \vec{V}_1| \overbrace{\left|\frac{d\sigma}{d\Omega}\right| d\Omega}^{b\, db\, d\phi}$$

Almost done!



$$\left(\frac{\partial f_1}{\partial t}\right)_{coll.} = \int_{|\vec{r}_1 - \vec{r}_2| \le d} d^3\vec{r}_2\, d^3\vec{p}_2\, (\vec{V}_1 - \vec{V}_2) \cdot \frac{\partial f_2}{\partial \vec{r}}$$

$$= \int d^3\vec{r}_2\, d^3\vec{p}_2\, |\vec{V}_1 - \vec{V}_2| \frac{\partial f_2}{\partial X}$$

$$= \int d^3\vec{p}_2 \int \underbrace{b\, db\, d\phi\, |\vec{V}_1 - \vec{V}_2|}_{\omega} \underbrace{\int \frac{\partial f_2}{\partial X} dX}_{f_2(X_2) - f_2(X_1)}$$

$$= \int d^3\vec{p}_2\, d^3\vec{p}_1'\, d^3\vec{p}_2'\, \omega(\vec{p}, \vec{p}_2 | \vec{p}_1', \vec{p}_2') \left[f_2(X_2) - f_2(X_1) \right]$$

Molecular chaos:
$$\begin{cases} f_2(X_1) = f_1(\vec{r}, \vec{p})\, f_1(\vec{r}, \vec{p}_2) \\ f_2(X_2) = f_1(\vec{r}, \vec{p}_1')\, f_1(\vec{r}, \vec{p}_2') \end{cases}$$

$$\boxed{\left(\frac{\partial f_1}{\partial t}\right)_{coll.} = \int d^3\vec{p}_2\, d^3\vec{p}_1'\, d^3\vec{p}_2' \;\; \omega(\vec{p},\vec{p}_2|\vec{p}_1',\vec{p}_2')\left[f_1(\vec{r},\vec{p}_1')f_1(\vec{r},\vec{p}_2') - f_1(\vec{r},\vec{p})f_1(\vec{r},\vec{p}_2)\right]}$$

Boltzmann equation