Review :

1 - Particle distribution function : $f_1(\vec{r},\vec{P}) := N\int \prod_{i=2}^{N} d^3\vec{r}_i\, d^3\vec{P}_i\; f(\{\vec{r}\},\{\vec{P}\})$

evolution : $\frac{\partial f_1}{\partial t} = \{H_1, f_1\} + \left(\frac{\partial f_1}{\partial t}\right)_{coll.}$

$$\left(\frac{\partial f_1}{\partial t}\right)_{coll.} = \int d^3\vec{r}_2\, d^3\vec{P}_2\; \frac{\partial U(\vec{r}-\vec{r}_2)}{\partial \vec{r}} \cdot \frac{\partial f_2}{\partial \vec{P}}$$

$$f_2(\vec{r},\vec{r}_2,\vec{p},\vec{P}_2) := N(N-1)\int \prod_{i=3}^{N} d^3\vec{r}_i\, d^3\vec{P}_i\; f(\{\vec{r}\},\{\vec{P}\})$$

$$n(\vec{r}) = \int f_1(\vec{r},\vec{P})\, d^3\vec{P}$$

$$\frac{\partial n}{\partial t} = \int \frac{\partial f_1}{\partial t}\, d^3\vec{P} = \int \left(\{H_1, f_1\} + \int d^3\vec{r}_2\, d^3\vec{P}_2\; \frac{\partial U(\vec{r}-\vec{r}_2)}{\partial \vec{r}} \cdot \frac{\partial f_2}{\partial \vec{P}}\right) d^3\vec{P}$$

$$\frac{\partial n}{\partial t} = \int \{H_1, f_1\}\, d\vec{P} + \underbrace{\int d^3\vec{r}_2\, d^3\vec{P}_2\; \frac{\partial U(\vec{r}-\vec{r}_2)}{\partial \vec{r}} \cdot \underbrace{\left(\int \frac{\partial f_2}{\partial \vec{P}}\, d^3\vec{P}\right)}_{0}}_{0}$$

$$\boxed{\frac{\partial n}{\partial t} = \int \{H_1, f_1\}\, d^3\vec{P}}$$

independent of Collision integral !

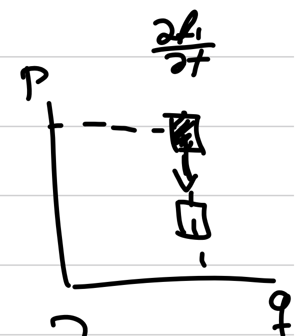
Boltzmann equation :

$$\frac{\partial f_1}{\partial t} = \{H_1, f_1\} + \left(\frac{\partial f_1}{\partial t}\right)_{coll.}$$

$$\left(\frac{\partial f_1}{\partial t}\right)_{coll.} = \int d^3\vec{r}_2\, d^3\vec{P}_2\; \frac{\partial U(\vec{r}-\vec{r}_2)}{\partial \vec{r}} \cdot \frac{\partial f_2}{\partial \vec{P}}$$

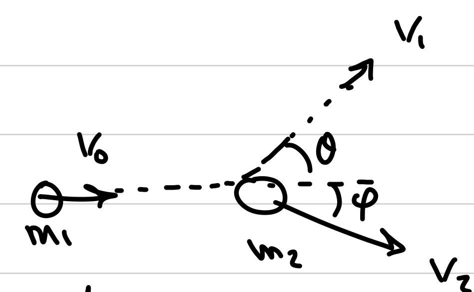
$\left(\frac{\partial f_1}{\partial t}\right)_{Coll.}$

$$\text{Rate} = \omega(\vec{p},\vec{p}_2 \mid \vec{p}_1{}',\vec{p}_2{}')\, f_2(\vec{r},\vec{r},\vec{p},\vec{p}_2)\, d^3\vec{p}_2\, d^3\vec{p}_1{}'\, d^3\vec{p}_2{}'$$



$$\left(\frac{\partial f_1}{\partial t}\right)_{Coll.} = \int d^3\vec{p}_2\, d^3\vec{p}_1{}'\, d^3\vec{p}_2{}' \Big[\omega(\vec{p}_1{}',\vec{p}_2{}' \mid \vec{p},\vec{p}_2)\, f_2(\vec{r},\vec{r},\vec{p}_1{}',\vec{p}_2{}') - \omega(\vec{p},\vec{p}_2 \mid \vec{p}_1{}',\vec{p}_2{}')\, f_2(\vec{r},\vec{r},\vec{p},\vec{p}_2) \Big]$$

Properties of $\omega(\vec{p},\vec{p}_2 \mid \vec{p}_1{}',\vec{p}_2{}')$:



equation	unknowns	
3	4	: 2D
4	6	. 3D

Conservation of Energy: $p^2 + p_2^2 = p_1'^2 + p_2'^2$

" " Momentum: $\vec{p} + \vec{p}_2 = \vec{p}_1{}' + \vec{p}_2{}'$

Symmetries:

- time-reversal:

$$\omega(\vec{p},\vec{p}_2 \mid \vec{p}_1{}',\vec{p}_2{}') = \omega(-\vec{p}_1{}',-\vec{p}_2{}' \mid -\vec{p},-\vec{p}_2)$$

- Parity: $(\vec{r},\vec{p}) \to (-\vec{r},-\vec{p})$

$$\omega(\vec{p},\vec{p}_2 \mid \vec{p}_1{}',\vec{p}_2{}') = \omega(-\vec{p},-\vec{p}_2 \mid -\vec{p}_1{}',-\vec{p}_2{}')$$

$$\Rightarrow \omega(\vec{p},\vec{p}_2 \mid \vec{p}_1{}',\vec{p}_2{}') = \omega(\vec{p}_1{}',\vec{p}_2{}' \mid \vec{p},\vec{p}_2)$$

$$\left(\frac{\partial f_1}{\partial t}\right)_{Coll.} = \int d^3\vec{p}_2\, d^3\vec{p}_1{}'\, d^3\vec{p}_2{}'\; \omega(\vec{p},\vec{p}_2 \mid \vec{p}_1{}',\vec{p}_2{}') \Big[f_2(\vec{r},\vec{r},\vec{p}_1{}',\vec{p}_2{}') - f_2(\vec{r},\vec{r},\vec{p},\vec{p}_2) \Big]$$

$$* \; f_2(\vec{r},\vec{r},\vec{p},\vec{p}_2) = f_1(\vec{r},\vec{p})\, f_1(\vec{r},\vec{p}_2) \; * \;\to\; \text{Assumption}$$

molecular chaos

$$\boxed{\left(\frac{\partial f_1}{\partial t}\right)_{Coll.} = \int d^3\vec{p}_2\, d^3\vec{p}_1{}'\, d^3\vec{p}_2{}'\; \omega(\vec{p},\vec{p}_2 \mid \vec{p}_1{}',\vec{p}_2{}') \Big[f_1(\vec{r},\vec{p}_1{}')\, f_1(\vec{r},\vec{p}_2{}') - f_1(\vec{r},\vec{p})\, f_1(\vec{r},\vec{p}_2) \Big]}$$

Boltzmann Equation

Detailed Balance :

$V = 0 \rightarrow H_1 = p^2/2m$

$$\frac{\partial f_1}{\partial t} = 0 \longrightarrow \underbrace{\{H_1, f_1\}}_{0} + \left(\frac{\partial f_1}{\partial t}\right)_{coll.} = 0$$

$f(\vec{p})$

$$\left(\frac{\partial f_1}{\partial t}\right)_{coll.} = 0 \longrightarrow f_1(\vec{r}, \vec{p}_1') f_1(\vec{r}, \vec{p}_2') - f_1(\vec{r}, \vec{p}) f_1(\vec{r}, \vec{p}_2) = 0$$

"detailed balance"

$$f_1(\vec{r}, \vec{p}_1') f_1(\vec{r}, \vec{p}_2') = f_1(\vec{r}, \vec{p}) f_1(\vec{r}, \vec{p}_2)$$

$$\underbrace{\ln f_1(\vec{p}_1') + \ln f_1(\vec{p}_2')}_{\substack{\sum \ln f \\ \text{after collision}}} = \underbrace{\ln f_1(\vec{p}) + \ln f_1(\vec{p}_2)}_{\substack{\sum \ln f \\ \text{before collision}}}$$

$$\ln f_1 = -\beta\left(\mu + E(\vec{p}) + \vec{u} \cdot \vec{p}\right)$$

$$E(p) = \frac{1}{2} m v^2 \quad ; \quad \vec{p} = m\vec{v}$$

$$\ln f_1 = -\beta\left(\mu + \frac{1}{2} m v^2 + m\vec{u} \cdot \vec{v}\right) = -\beta\left(\mu - \frac{1}{2} m u^2 + \frac{1}{2} m (\vec{v} - \vec{u})^2\right)$$

$$\boxed{f_1(\vec{v}) = A\, e^{-\beta\left(\frac{1}{2} m (\vec{v} - \vec{u})^2\right)}} \qquad \beta = \frac{1}{kT} \quad ; \quad \vec{u} : \text{Possible drift}$$