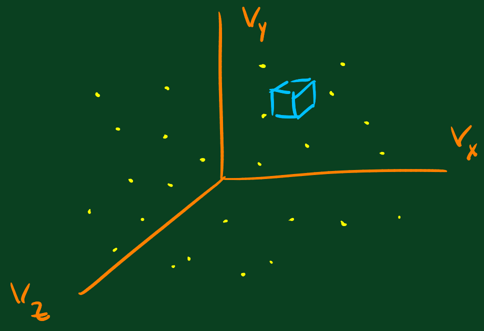
1. Basic Concepts

2. KT : $\begin{cases} \text{BBGKY} \\ \text{Boltzmann eq.} \end{cases}$

3. Stochastic Processes

4. linear response

Velocity distribution :



$$v \in (v, v+dv) \longrightarrow dN_v$$

$$\left.\begin{aligned} dN_{v_x} &= Nf(v_x)\, dv_x \\ dN_{v_y} &= Nf(v_y)\, dv_y \\ dN_{v_z} &= Nf(v_z)\, dv_z \end{aligned}\right\}$$

$$\frac{d^3N}{N} = f(v_x) f(v_y) f(v_z) \underbrace{dv_x\, dv_y\, dv_z}_{d^3\vec{v}}$$

$$= \varphi(v)\, d^3\vec{v}$$

$$f(v_x) f(v_y) f(v_z) = \varphi(v)$$

$$\ln f(v_x) + \ln f(v_y) + \ln f(v_z) = \ln \varphi(v)$$

$\partial/\partial v_x$

$$\frac{f'(v_x)}{f(v_x)} = \frac{\varphi'(v)}{\varphi(v)} \frac{\partial v}{\partial v_x}$$

$; \quad v^2 = v_x^2 + v_y^2 + v_z^2 \xrightarrow{\partial/\partial v_x} 2v \frac{\partial v}{\partial v_x} = 2v_x$

$$\longrightarrow \frac{\partial v}{\partial v_x} = \frac{v_x}{v}$$

$$\frac{1}{v_x} \frac{f'(v_x)}{f(v_x)} = \frac{1}{v} \frac{\varphi'(v)}{\varphi(v)} = \text{ثابت} = -\alpha$$

$$\frac{\varphi'(v)}{\varphi(v)} = -\alpha v \longrightarrow \int \frac{d\varphi}{\varphi} = -\alpha \int v\, dv \rightarrow \underline{\varphi(v) = A\, e^{-\frac{\alpha}{2} v^2}}$$

$$\frac{f'(v_x)}{f(v_x)} = -\alpha v_x \longrightarrow \text{"} \longrightarrow \text{"} \longrightarrow f(v_x) = B\, e^{-\frac{\alpha}{2} v_x^2}$$

$$f(v_x)\, f(v_y)\, f(v_z) = \varphi(v) \longrightarrow B^3\, \cancel{e^{-\frac{\alpha}{2}(v_x^2+v_y^2+v_z^2)}} = A\, \cancel{e^{-\frac{\alpha}{2} v^2}}$$

$$\underline{\underline{A = B^3}}$$

$$\begin{cases} \varphi(v) = A\, e^{-\frac{\alpha}{2} v^2} \\ f(v_x) = A^{1/3}\, e^{-\frac{\alpha}{2} v_x^2} \end{cases} \quad ; \quad \frac{d^3N}{N} = \varphi(v)\, d^3\vec{v}$$

<u>Normalization:</u> $\int \varphi(v)\, d^3\vec{v} = 1 \longrightarrow A \int e^{-\frac{\alpha}{2} v^2} \underbrace{d^3\vec{v}}_{v^2 \sin\theta\, dv\, d\theta\, d\phi} = 1$

$$1 = 2\pi A \int_0^\infty v^2\, e^{-\frac{\alpha}{2} v^2}\, dv = 2\pi A \left(-2 \frac{\partial}{\partial \alpha} \underbrace{\int e^{-\frac{\alpha}{2} v^2}\, dv}_{\sqrt{2\pi/\alpha}} \right)$$

$$\longrightarrow A = \left(\frac{\alpha}{2\pi} \right)^{3/2}$$

$$\underline{\varphi(v) = \left(\frac{\alpha}{2\pi} \right)^{3/2} e^{-\frac{\alpha}{2} v^2}} \quad ; \quad \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

$$\langle v^2 \rangle = \frac{3kT}{m} \quad ; \quad \underline{f(v) = \sqrt{\frac{\alpha}{2\pi}}\, e^{-\frac{\alpha}{2} v^2}}$$

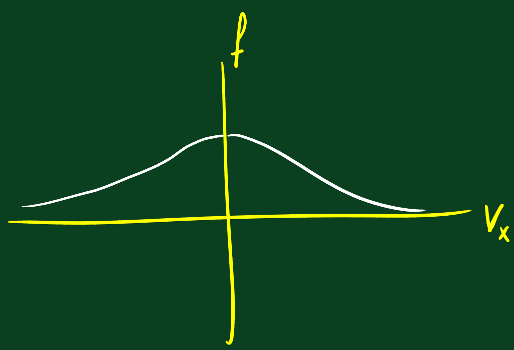
$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle \quad ; \quad v^2 = \sum_i v_i^2 \longrightarrow \langle v^2 \rangle = 3 \langle v_x^2 \rangle$$

$$\langle v^2 \rangle = \int v^2 \varphi(v)\, d^3\vec{v} = \left(\frac{\alpha}{2\pi}\right)^{3/2} \int v^2 e^{-\frac{\alpha}{2}v^2} \underbrace{d^3\vec{v}}_{v^2 \sin\theta\, dv\, d\theta\, d\phi}$$

$$= \cdots \int v^4 e^{-\frac{\alpha}{2}v^2} dv$$

$$A = \left(\frac{\alpha}{2\pi}\right)^{3/2} = \left(\frac{m}{2\pi kT}\right)^{3/2} \quad ; \quad \alpha = \frac{m}{kT}$$

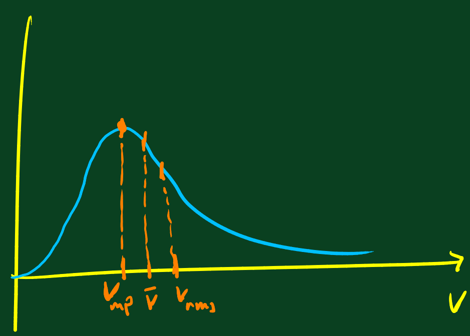
$$\begin{cases} \varphi(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} \\ f(v_x) = \sqrt{\frac{m}{2\pi kT}}\, e^{-\frac{mv_x^2}{2kT}} \end{cases}$$



$$dN(v, v+dv) \neq \varphi(v)\, dv$$

$$\frac{d^3N}{N} = \varphi(v)\, d^3\vec{v} = \varphi(v)\, v^2 \sin\theta\, dv\, d\theta\, d\phi$$

$$\frac{dN(v, v+dv)}{N} = 2\pi v^2 \varphi(v)\, dv$$

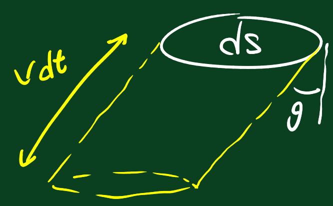


$$v_{mp} : \bar{v} : v_{rms}$$

$$1 : 1.13 : 1.22$$

Flux:

$$\frac{dN_{\theta,\phi}}{N} = \frac{\cancel{r^2}\sin\theta\, d\theta\, d\phi}{4\pi \cancel{r^2}} = \frac{\sin\theta\, d\theta\, d\phi}{4\pi}$$



$$dN_{\theta,\phi,v;ds} = dN_v\left(\frac{\sin\theta\, d\theta\, d\phi}{4\pi}\right)\left(\frac{ds\; v\,dt\cos\theta}{V}\right)$$

$$\Phi := \frac{dN_{ds}}{ds\,dt} = \frac{v\,dN_v}{\underset{2}{\cancel{4\pi}}\, V}\underbrace{\int_0^{\pi/2}\sin\theta\cos\theta\, d\theta}_{1/2} \times \cancel{2\pi} = \frac{1}{4}\,\frac{1}{V}\underbrace{\int v\, dN_v}_{N\bar{v}}$$

$$\bar{v} = \frac{v_1\Delta N_1 + v_2\Delta N_2 + \cdots}{N} = \frac{\int v\, dN_v}{N}$$

$$\Phi = \frac{1}{4}\frac{N}{V}\bar{v} \quad ; \quad n := \frac{N}{V} \implies \boxed{\Phi = \frac{1}{4} n\bar{v}}$$

Pressure:

$$dP = 2mv\cos\theta\; dN_{\theta,\phi;ds} \quad ; \quad P = \frac{dP}{ds\,dt}$$

$$P = \frac{m}{\cancel{2\pi} V}\underbrace{\int v^2\, dN_v}_{N\langle v^2\rangle}\int_0^{\pi/2}\underbrace{\cos^2\theta\,\sin\theta\, d\theta}_{1/3} \times \cancel{2\pi} = \frac{1}{3} m \frac{N}{V}\langle v^2\rangle$$

$$\left.\begin{aligned} PV &= N\left(\frac{1}{3}m\langle v^2\rangle\right) \\ PV &= NkT \end{aligned}\right\} \quad \boxed{\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT}$$

$$U = N\left\langle \frac{1}{2}mv^2\right\rangle = \frac{1}{2}mN\langle v^2\rangle$$

$$\longrightarrow \boxed{U = \frac{3}{2}PV}$$