Represented by S. Maleki

Historical Papers

On the Theory of the Energy Distribution Law of the Normal Spectrum

by M. Planck, 1900

The Presentation Contents

The Black Body Radiation (BBR) Problem

First attempts

- Eugen Lommel (1837 – 1899): Mechanical model of vibrations in solid (Lommel, 1878)
- Vladimir Alexandrovich Michelson (1860 – 1927): Kinetic Theory (Michelson, 1888)
- Ludwig Boltzmann (1844 – 1906): Electromagnetism (Boltzmann, 1884a & Boltzmann, 1884b)

❖ Boltzmann used a connection between radiation and the second law of thermodynamics which had been put forward several years earlier by the Italian physicist Adolfo Bartoli (Bartoli, 1876)

What Did People Know?

Empirical and Theoretical Backgrounds

1. Stefan's Law (1879)

$$P = \sigma T^4$$

P: Total Radiative Power

$\sigma \approx 5.67 \times 10^{-8}$ watts/sq.m./K^4

"derived theoretically by L. Boltzmann five years later (Boltzmann, 1884b)"

2. Wien's Displacement Law (1893)

$$f(I_{max}) \propto T$$

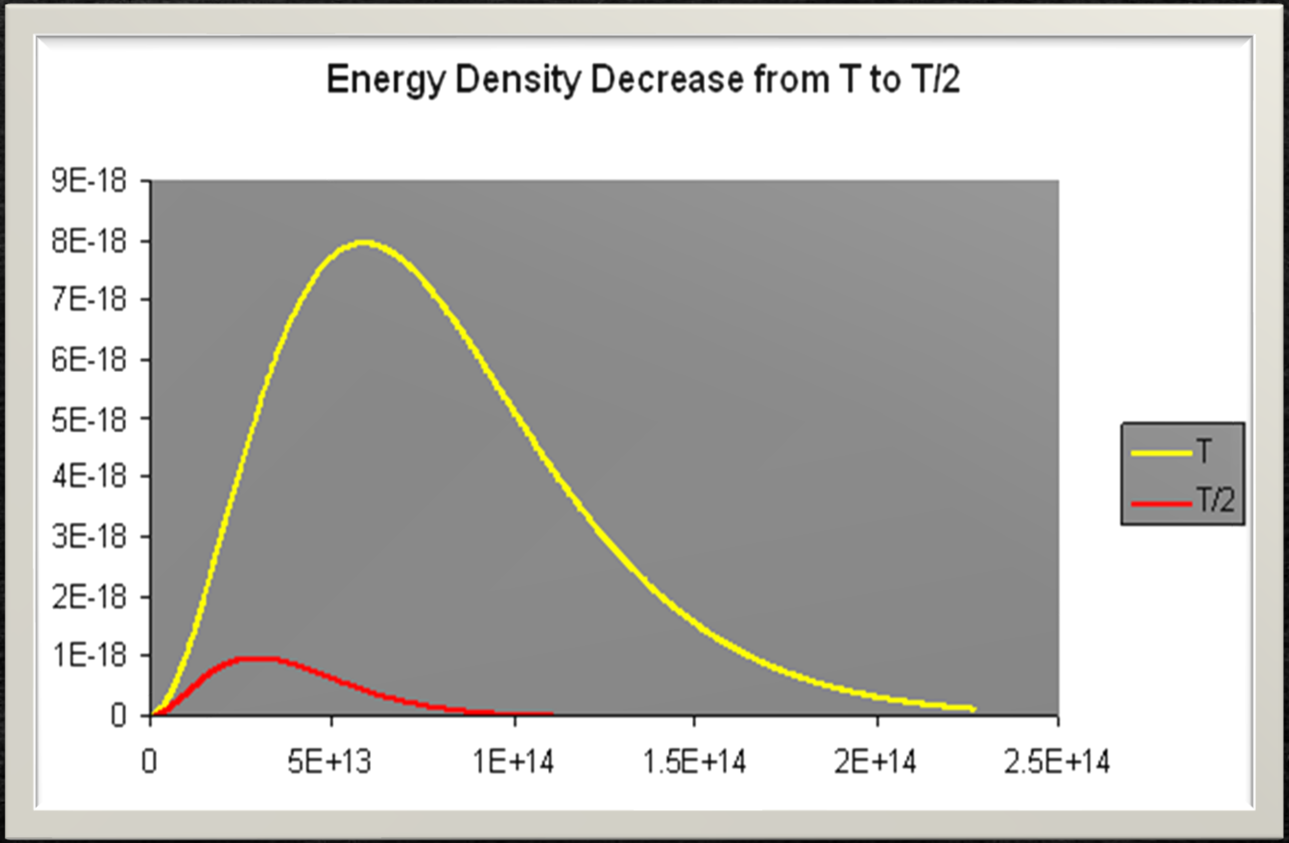
Deduced theoretically by W. Wien, following Boltzmann's thermodynamic reasoning.

"It had been observed semi-quantitatively by an American astronomer, S. Langley"

$$T\lambda = T_0\lambda_0$$

Wien & Lummer, 1895: Empirical Radiation Intensity/Frequency Curve

- They confirmed Stefan's law
- They confirmed Wien's displacement law



"This is the correct plot

- Paschen, 1896: $$\Phi(\lambda, T) = c_1 \lambda^{-\alpha} \exp\left(-\frac{c_2}{\lambda T}\right)$$
  where $\alpha \approx 5.5$

- Wien, 1896: $$\rho_\lambda = F(\lambda) \exp\left(-\frac{f(\lambda)}{T}\right)$$
  + Displacement law:
  $$f(\lambda) = \frac{c_2}{\lambda}$$
  + Stefan's law:
  $$F(\lambda) = \frac{c_1}{\lambda^5}$$

Max Planck Entered the Game

Max Planck Entered the Game

- First paper concerning the properties of electromagnetic radiation (Planck, 1895a):
- Treatment of the damping effect arising from the emission of radiation (Planck, 1896)

He explained the goal of his endeavours in the following words:

The study of conservative damping appears to me to be of fundamental importance due to the fact that through it one's view is opened towards the possibility of a general explanation of irreversible processes with the help of conservative forces—a problem which confronts the theoretical research in physics more urgently every day. (Planck, 1896, p. 154)

Max Planck Entered the Game

- Planck didn't like at all the probabilistic nature of entropy (introduced by Boltzmann)

such a resonator will be excited by absorbing energy from the radiation incident upon it from the outside, and it will be damped by emitting energy. Now the emitted energy will not, in general, be of the same type as the absorbed energy; hence the resonator will change by its vibration the nature of the electromagnetic waves propagating through its vicinity to some extent. It can be shown that these changes possess, in several respects, a certain direction, i.e., the tendency to homogenize. (Planck, 1897a, p. 59)

Planck's Attempts

His previous contributions to the BBR problem and the heat radiation

Planck's Attempts

- Five contributions entitled " Über irreversible Strahlungsvorgänge" (On Irreversible Radiation Processes) between Feb1897-May1899 (Planck, 1897a, b, c, 1898, 1899)
- These investigations resulted in establishing Wien's law as the law defining the energy distribution of BBR.

Planck's Attempts

- (Planck, 1897a): Calculated the effect of a resonator (having a large wavelength and small damping constant) on incident electric waves
- (Planck, 1897c): After rejecting a criticism of Boltzmann, who claimed that all processes used by planck were reversible, he went on to prove the existence of irreversibility in a system in which a resonator is situated in the centre of a spherically shaped cavity.

"However, in carrying out the proof, he had to make an assumption concerning the nature of radiation"

- (Planck, 1898): Introducing the concept of "natural radiation" as a hypothesis (defining entropy)
- (Planck, 1899): Generalized his proof of irreversibility to a cavity containing radiation and many resonators. He also identified the "electromagnetic" and "thermodynamic" entropy and derived BBR law.

Planck's Attempts

Within less than three years, Planck achieved his goal of connecting the thermodynamic and electrodynamic theories. Especially he introduced the concept of irreversibility for resonators in cavity.

But, he had to pay a price for this success: The Hypothesis of Natural Radiation.

In 1899 Planck was aware of the fact that:

The hypothesis of natural radiation, if assumed to be valid for all space points at all times, implies at its core the second law of thermodynamics when applied to the processes of radiation: that is, it is another expression of the same law. (Planck, 1900a, p. 74)

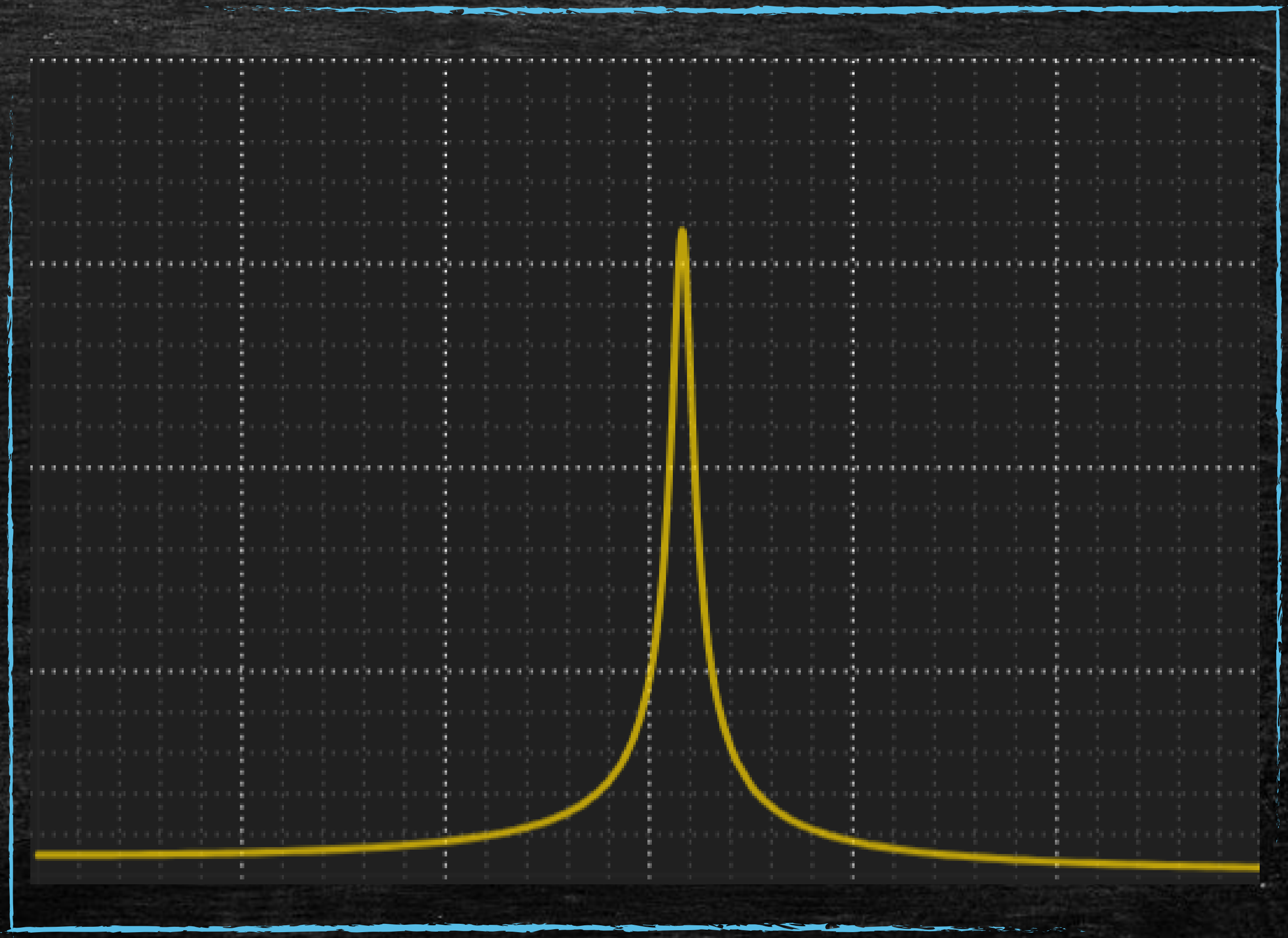
Planck's Procedure

$$m\ddot{x} + m\omega_0^2 x^2 - \frac{2e^2}{3c^3}\dddot{x} = eE\ cos(\omega t)$$

$$A = \frac{eE}{\sqrt{m^2(\omega^2 - \omega_0^2)^2 + (\gamma\omega)^2}}$$

$$U = \frac{1}{2}m\omega_0^2 A^2$$

Planck's Procedure



Amplitude of oscillations in term of incident wave frequency

Planck's Procedure

$$m\ddot{x} + m\omega_0^2 x^2 - \frac{2e^2}{3c^3}\dddot{x} = eE\,cos(\omega t)$$

$$A = \frac{eE}{\sqrt{m^2(\omega^2 - \omega_0^2)^2 + (\gamma\omega)^2}}$$

$$U = \frac{1}{2} m\omega_0^2 A^2$$

$$U \approx \frac{1}{8m} \frac{e^2}{(\omega_0 - \omega)^2 + (\gamma/2m)^2} E^2$$

Planck's Procedure

$$U \approx \frac{1}{8m} \frac{e^2}{(\omega_0 - \omega)^2 + \left(\frac{\gamma}{2m}\right)^2} E^2$$

$$\int_{-\infty}^{\infty} \frac{1}{x^2 + a^2}\, dx = \frac{\pi}{a}$$

$$U = \frac{e^2}{8m} \frac{2\pi m}{\gamma} E^2 = \frac{3\pi c^3}{8\omega_0^2} E^2$$

$$E^2 = \frac{8\omega_0^2}{3\pi c^3} U$$

Planck's Procedure

$$\rho(f,T) = \left(\frac{8\pi f^2}{c^3}\right) U(f,T)$$

Wien's law:

$$\rho(f,T) = \alpha f^3 \exp\left(-\frac{\beta f}{T}\right)$$

$$U(f) = \frac{\alpha c^3 f}{8\pi} \exp\left(-\frac{\beta f}{T}\right)$$

Planck's Procedure

Planck focuses on entropy:

$$dS = \frac{dU}{T}$$

$$S = -\frac{U}{\beta f}\left(\ln\left(\frac{8\pi U}{\alpha f c^3}\right) - 1\right)$$

$$\frac{\partial^2 S}{\partial U^2} = -\frac{1}{\beta f U}$$

New Experimental Data

(Lummer & Jahnke, 1900)

New Experimental Data

- October 1900, Rubens and Kurlbaum

$$\rho_\lambda \propto T$$

In agreement with formulae derived earlier by Lord Rayleigh (1900b) and Lummer & Jahnke (1900)

$$U = kT$$

Equipartition was holding!

$$\frac{\partial^2 S}{\partial U^2} = -\frac{k}{U^2}$$

Planck Made A Guess!

Abandoning thoughts of deep thermodynamic truths, he decided he'd better patch things up as best he could

Planck Made A Guess!

$$\frac{\partial^2 S}{\partial U^2} = -\frac{k}{U(hf + U)}$$

$$S = k\left(1 + \frac{U}{hf}\right)\ln\left(1 + \frac{U}{hf}\right) - k\left(\frac{U}{hf}\right)\ln\left(\frac{U}{hf}\right)$$

$$U = \frac{hf}{\exp(hf/kT) - 1}$$

$$\rho(f,T) = \frac{8\pi f^2}{c^3}\frac{hf}{\exp(hf/kT) - 1}$$

Looking For A Theory…

Looking For A Theory...

- To explain the *Ansatz* for the entropy function, it was evidently necessary to consider several resonators in the cavity.
- Entropy function depends not only on the total energy of all resonators but also on the distribution of energy among the individual resonators
- To treat emitters like molecules (as Wien did in 1896). But in order to obtain a complete theory, more features are needed.

Planck turned to "Boltzmann's method"…

Planck turned to "Boltzmann's method"...

- Planck had withdrawn his earlier criticism and increasingly approached Boltzmann's view on the origin of irreversibility, especially by introducing the hypothesis of "natural radiation" and recognizing the similarity of this concept with that of "molecular chaos" which Boltzmann had introduced in kinetic theory.
- In fall 1900 Planck was prepared well enough and willing to search in Boltzmann's statistical theory of matter. He recalls in his Nobel lecture:

This problem led me automatically to a consideration of the connection between entropy and probability, that is, to Boltzmann's trends of ideas, until after some weeks of the most strenuous work of my life, light came into the darkness and a new perspective began to open up for me (Planck, 1920, p. 5)

Planck turned to "Boltzmann's method"...

- Planck formulated Boltzmann's relation as

$$S = k \ln \Omega$$

where Ω is the total number of complexions (permutation measure).

$$S = k\left(1 + \frac{U}{hf}\right)\ln\left(1 + \frac{U}{hf}\right) - k\left(\frac{U}{hf}\right)\ln\left(\frac{U}{hf}\right)$$

- Planck studied in detail Boltzmann's great memoir, and he discovered right away the desired expression in its first section (Boltzmann, 1877)
- There Boltzmann had given an equation for J, the total number of complexions or possibilities of distributing λ discrete, equal energy values ϵ among n molecules as

$$J = \frac{(\lambda + n - 1)!}{\lambda!\,(n-1)!}$$

Planck turned to "Boltzmann's method"...

$$J = \frac{(\lambda + n - 1)!}{\lambda!\,(n-1)!}$$

$$J = \frac{1}{\sqrt{2\pi}} \frac{(\lambda + n - 1)^{\lambda + n - \frac{1}{2}}}{(n-1)^{n - \frac{1}{2}}\,\lambda^{\lambda + \frac{1}{2}}}$$

$$\ln(J) = n\left[\left(\frac{\lambda}{n} + 1\right)\ln\left(\frac{\lambda}{n} + 1\right) - \frac{\lambda}{n}\ln\left(\frac{\lambda}{n}\right)\right]$$

Planck turned to "Boltzmann's method"...

$$\ln(J) = n\left[\left(\frac{\lambda}{n}+1\right)\ln\left(\frac{\lambda}{n}+1\right)-\frac{\lambda}{n}\ln\left(\frac{\lambda}{n}\right)\right]$$

$$S = k\left(1+\frac{U}{hf}\right)\ln\left(1+\frac{U}{hf}\right)-k\left(\frac{U}{hf}\right)\ln\left(\frac{U}{hf}\right)$$

$$\frac{\lambda}{n}=\frac{U}{hf}$$

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